, where D = drag

Velocity deficit created by the B.L. is the physical origin of the drag force!

Von Karman defined this relationship in 1921!

Our Goals for Today (lect. 6)

1. A very fast review of derivation of Bernoulli eq. with emphasis on non-steady form of the equation
2. Invitation to self-study notes for Sec. 3.6 and 3.7 (angular momentum and Energy formulations of RTT)
3. Starting Chapter 4 and introducing differential forms of conservation of mass & momentum

Diagram

Description automatically generatedSec. 3.5 Bernoulli Equation

Will not consider shear stress or the wall, So ONLY frictionless flow

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Writing RTT for momentum Equation (2):

An element of an infinite section:



– along the streamline

- integrate over starred variable

- integrate over starred variable

à constant in time

à

à

à

Where à

Due to mass conservation, these terms cancel each other, see Eq. (1) : ,

On the other hand, forces on stream tube element are pressure & gravity so,

à gauge pressure only; for a small angled tube, approx. dPdA

A picture containing text, whiteboard

Description automatically generated

Equation (15) is unsteady frictionless along a stream tube (to write for a streamline, we put stream on a diet!)

If we consider flow is steady state, then

If we consider fluid to be incompressible (ρ = constant), Then I can simply write à decoupling P & ρ

So, integrating between 2 points, 1&2:

NOTE: In deriving Equations 15 & 16, we used conservation of mass & momentum but NOT conservation of Energy! à so, if heat or work are added/removed from c.v., Then equations (15&16) are not applicable. Do NOT use!

Observation:

From 16:

à pressure head

à dynamic head (velocity head)

à static head

H à Head

See Figure 3.13

Diagram

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2. If the change in Z is negligible, then Equation 16:

à static pressure

à dynamic pressure

à Total pressure (stagnation pressure)

Look at the exp. 3.16 in the textbook.

Sec. 3.6 – Angular Momentum Theorem:

In systems like pumps and turbines where flow rotates, the linear may not be very useful for analysis. Also, anywhere that there is a misalignment between fluid flow and force line of action.

Note a fluid is deformable unlike a solid, so angular momentum should be written on elemental basis:

So,

Rotation about point “o”

From RTT

From mechanisms, we know:

à summation of moments for all forces (gravity, pressure, etc.) around “o”

Most problems can be treated as 1D intel/outlet and analyzed in steady conditions.

So:

Otherwise, one must use a differential approach (see Chapter 4) and use computers to solve equations numerically.

Section 3.7 – Energy Equation

This is our 4th law to consider:

Energy (e) has several forms: , and many other chem., elec., …

Usually, we deal with the 1st 3 types:

* Work (w) can usually have 3 forms:
  + Wshaft
  + Wpressure à
  + Wviscosity à
* The heat (Q) has normally 3 forms: , but in this course we mainly deal with isothermal flows (no details for heat transfer)

Using RTT with

Where h = enthalpy = à pressure work brought from LHS to RHS

For steady, 1D flow:

Evaluate as an average over the in/out area

Diagram

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1. Pay attention to the how c.v. is selected to make the analysis a simple flow in à flow out problem.
2. No need to consider pressure as live action passes through “O”, so momentum is generated
3. The same as 2 is true for normal component of velocity vector
4. Assume 1D flow due to defined c.v., then use Eq. 18

à Torque

Steady flow:

Q à Flow rate

à where sin90 = 1; clockwise

From (1): à clockwise

Knowing that &

à clockwise; Euler equation of pump

Do part (b) on your own.

**Chapter 4**

Diagram

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Chapter 4: Differential Approach to fluid flow

Sec 4.1

Some math reminders!

Remember

But u = u (x, y, z, t)

Where

Aside: (Operator!)

Same way and can be found

Finally:

à local term

à convective term due to e.g., change in geometry. Note: particles of fluid have relative motion w.r.t. one another unlike solids.

So even a steady flow, can have acceleration because of convective term (e.g., steady flow through a converging nozzle)

A close-up of a logo

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How about Pressure?

P = P (x, y, z, t), so in general:

Aside: In an isotropic flow:

Let’s write on 4 laws to do differential analysis for flows:

Sec 4.2 Conservation of Mass

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NOTE: All other pair of sides will also have mass flow into/out of them, but for graphical clarity not shown

Writing similarity for z & y directions AND considering the mass charge over time within the element (i.e., ) that must be balanced to zero (to satisfy conservation of mass):

If the flow is incompressible, regardless of whether or not it is steady

Example: An inviscid flow through a 60˚ angle elbow. Here are the flowing velocity components

Where a & b are constants

If flow is incompressible, what is the velocity component v?

Solution: The flow everywhere should satisfy conservation of mass, momentum, etc. Looking at conservation of momentum, i.e., Equation 5 or 6, one can find v knowing u & w, so:

Where f (x, z, t) à arbitrary function

ASIDE: If the Mach number of flow is below 0.3, then any flow (gas or liquid) can be considered incompressible; so far air flow below ≈ 300km/hr in normal atmosphere is considered incompressible.

Let’s do the 2nd law in Fluid Mechanics: Conservation of Momentum

Chart

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What type of forces act on this element?

1. Change in momentum in any x, y, or z direction over the element
2. Body force (e.g., gravity, electrical, magnetic, etc.)
3. Surface Forces
   1. Hydrostatic pressure (P)
   2. Viscos stresses ()

1 Momentum Flux

Consider the x-direction

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Not momentum flux in s-direction:

Same can be done for the other two direction (i.e., y & z)

2 Gravity

Only in y-direction

Diagram

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3a Hydrostatic Pressure

Consider x-direction

**Chart

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Net force = (isotropic flow)

Negative direction can be seen by arrow in element.

The same treatment can be done for other directions.

3b Viscos Stresses

Consider x-direction

Diagram

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à normal stress due to shear forces

Net force due to shear in x-direction =

Same for other directions à a viscos stress term!

Consider 2nd law of Newton:

Let’s write it in the x-direction:

Where, à hydro-static pressure and is viscos.

In the y-direction:

Where, à due gravity

In the z-direction:

Equations (7-9) are general momentum (linear) equations in 3D

For inviscid flow:

Euler Equations for fluid flow

Equations (7-9) can be recasted for Newtonian fluids by relating shear to the rate of change of velocity in space (x, y, z), using the concept of viscosity.

The 3D treatment of viscosity for an incompressible fluid is:

Substituting above relationships for τ into equations 7-9, one has:

Where, , , and are the convective term & it is a non-linear differential equation.

* RHS if equations are the inertial terms

Equation (11) is called Navier-Stokes Equation for incompressible fluids.

Sec 4.4 Differential Equations of Angular Momentum (3rd flow)

Let’s consider the notation around centroid of an element (O, i.e., where z-axis crosses the x-y plane)

A white board with writing on it

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If I write the equation for moment calculation, I find that:

For rotation around y or x axes, one finds:

The above means that there is no differential equation form for conservation of angular momentum à one should use the integral forms given in chapter 3.

Note 1: P, , , and all pass through the centroid of the element (O) so they have momentum around O.

Note 2: Fluid similar to solids experiences symmetric shear stresses.

Sec. 4.4 Differential Equations for Conservation of Energy (4th Law)

It can be shown that the conservation of energy equation for an element will be equation (12):

à defined in chapter 3

Note: There is no infinitesimal shaft work (a mechanical shaft cannot be the size of an element)

* Need to find differential forms of and

1. Heat Transfer to the Element

To deal with , we only consider conduction (vast application only have conduction)

Fourier’s Law for Conduction

or in general (3D)